Scalar or Dot Product :

The scalar product of two vectors $\vec{a}$ and $\vec{b}$ is denoted by $\vec{a}\cdot\vec{b}$ is defined as the product of the magnitudes of $\vec{a}$ and and cosine of the angle between them.

i.e., $\vec{a}\cdot\vec{b} = |\vec{a}|\cdot|\vec{b}|\cos\theta$, θ is angle b/w $\vec{a}$ & $\vec{b}$.

Properties of Scalar Product

1) Scalar Product of two vectors is commutative.

$$\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}.$$

2) The necessary & sufficient condition for two non-zero vectors to be perpendicular is that their scalar product is zero

i.e., $\vec{a}\perp\vec{b}$ iff $\vec{a}\cdot\vec{b} = 0$

3) Scalar product of orthonormal vector triad $\vec{i}, \vec{j}, \vec{k}$ is also zero.

4) The scalar product of a vector with itself is the square of the modulus of that vector.

i.e., $\vec{a}\cdot\vec{a} = |\vec{a}|\cdot|\vec{a}|\cos 0$

$= a.a.1 = a^2$

5) If 'm' and 'n' be any scalar, then

$$m\vec{a}\cdot n\vec{b} = mn(\vec{a}\cdot\vec{b}) = (mn\vec{a})\cdot\vec{b} = \vec{a}\cdot(mn\vec{b})$$

6) The cosine of the angle b/w two vectors is the scalar product of the unit vectors in the directions of the given vectors. i.e., $\cos\theta = \hat{a}\cdot\hat{b}$

7) The scalar product of parallel vectors, or collinear vectors is :-

If $\vec{a}$ & $\vec{b}$ have same direction then $\vec{a}\cdot\vec{b} = ab$

If $\vec{a}$ & $\vec{b}$ have opposite directions then $\vec{a}\cdot\vec{b} = -ab$.

8) $\vec{a}\cdot(\vec{b}+\vec{c}) = \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c}$.

Vector or Cross Product

The vector or cross product of two vectors $\vec{a}$ and $\vec{b}$ is denoted by $\vec{a}\times\vec{b}$ and is defined as

$$\vec{a}\times\vec{b} = |\vec{a}|\cdot|\vec{b}|\sin\theta\cdot\hat{n}$$
$$= ab\sin\theta\,\hat{n}$$

Properties of Vector Product

1) Vector product of two vectors is anti-commutative i.e., $\vec{a}\times\vec{b} = -\vec{b}\times\vec{a}$

2) The necessary and sufficient condition for two non-zero vectors to be parallel is that their vector product should be zero.

3) The cross product of any vector with itself is zero i.e., $\vec{a}\times\vec{a} = \vec{0}$

4) Vector product of the orthogonal righ-handed vector triad $\vec{i}$, $\vec{j}$ and $\vec{k}$ is zero.

5) If 'm' and 'n' be any scalar, then, by definition

$$m\vec{a} \times n\vec{b} = mn(\vec{a} \times \vec{b}) = (mn\vec{a}) \times \vec{b} = \vec{a} \times (mn\vec{b})$$

Also $\vec{a} \times (-\vec{b}) = (-\vec{a}) \times \vec{b} = -(\vec{a} \times \vec{b})$

$$(-\vec{a}) \times (-\vec{b}) = \vec{a} \times \vec{b}$$

6) Distributive law:

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$$

Vector function of a Scalar Variable

Let $\vec{u}$ be a variable vector depending on scalar 't' which varies in the interval (α, β).

Then $\vec{u}$ is a vector function of the scalar 't' and is denoted as $\vec{u} = \overrightarrow{f(t)}$ where 'f' denotes the law of correspondence determining the magnitude as well as direction of $\vec{u}$.

The vector $\overrightarrow{f(t)}$ is also written as $\vec{f}(t)$.

Note: If $\vec{r}$ is the position vector of any point on the curve, the equation of a space curve is given by

$$\vec{r} = f_1(t)\vec{i} + f_2(t)\vec{j} + f_3(t)\vec{k}$$

where

$$x = f_1(t) \, ; \, y = f_2(t) \, ; \, z = f_3(t)$$

where t is the parameter.

Limit of a Vector Function

Defn: A vector function $\vec{f}(t)$ is said to tend to a vector limit $\vec{l}$ as $t \to a$, if corresponding to any positive number 'ε' that we may choose, no matter how small, there exists a positive number 'δ' such that

$$|\vec{f}(t) - \vec{l}| < \varepsilon \quad \text{for } |t-a| \leq \delta$$

i.e., $$\lim_{t \to a} \vec{f}(t) = \vec{l}$$

We also say that $\vec{f}(t)$ tends to $\vec{l}$ as t tends to 'a'; and express as $\vec{f}(t) \to \vec{l}$ as $t \to a$

Fundamental Theorems on Limits.

If $\vec{f}(t)$, $\vec{g}(t)$ be vector functions and $\phi(t)$ a scalar function of 't' such that as $t \to a$.

$$\vec{f}(t) \to \vec{l}, \quad \vec{g}(t) \to \vec{m} \text{ and } \phi(t) \to n$$

then (i) $\vec{f}(t) + \vec{g}(t) \to \vec{l} + \vec{m}$, as $t \to a$.

(ii) $\vec{f}(t) - \vec{g}(t) \to \vec{l} - \vec{m}$, as $t \to a$

(iii) $\phi(t)\vec{f}(t) \to n\vec{l}$ as $t \to a$

(iv) $\vec{f}(t) \cdot \vec{g}(t) \to \vec{l} \cdot \vec{m}$ as $t \to a$

(v) $\vec{f}(t) \times \vec{g}(t) \to \vec{l} \times \vec{m}$, as $t \to a$